THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH3070 Introduction to Topology 2017-2018
Tutorial Classwork 7

1. Let $(X, \mathfrak{T})$ be a topological space. Let $\{F_i\}_{i\in I}$ be a collection of closed subsets and $U \in \mathfrak{T}$ such that $\cap_{i\in I} F_i \subset U$.

   (a) Assume that X is compact. Show that there exists $i_1, i_2, \ldots, i_n \in I$ such that $\cap_{k=1}^{n} F_{i_k} \subset U$;

   (b) Assume that F_{i_0} is compact for some $i_0 \in I$. Show that there exists $i_1, i_2, \ldots, i_n \in I$ such that $\cap_{k=1}^{n} F_{i_k} \subset U$.

2. Let $(X, \mathfrak{T})$ be a compact space.

   (a) Consider a sequence of non-empty closed subsets $\{F_n\}_{n\in\mathbb{N}}$ with $F_{n+1} \subset F_n$ for $n \geq 1$. Show that we have $\cap_{n\in\mathbb{N}} F_n \neq \emptyset$.

   (b) * Further assume that X is Hausdorff. Show that for any continuous function $f : X \to X$, there exists a non-empty closed subset $F \subset X$ such that $f(F) = F$.

   (Hint: consider the sequence $F_1 = X, F_{n+1} = f(F_n)$ for $n \geq 1$.)